

## Analyze and Use Proportional Relationships

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TOPIC  
2

### ANALYZE AND USE PROPORTIONAL RELATIONSHIPS

#### ? Topic Essential Question

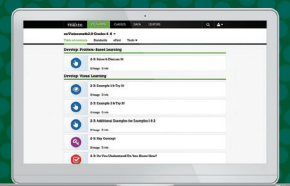
How can you recognize and represent proportional relationships and use them to solve problems?

#### Topic Overview

- 2-1 Connect Ratios, Rates, and Unit Rates
- 2-2 Determine Unit Rates with Ratios of Fractions
- 2-3 Understand Proportional Relationships: Equivalent Ratios
- 2-4 Describe Proportional Relationships: Constant of Proportionality
- 3-Act Mathematical Modeling: Mixin' It Up
- 2-5 Graph Proportional Relationships
- 2-6 Apply Proportional Reasoning to Solve Problems

#### Topic Vocabulary

- constant of proportionality
- proportion
- proportional relationship



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#### Lesson Digital Resources

- INTERACTIVE ANIMATION** Interact with visual learning animations.
- ACTIVITY** Use with *Solve & Discuss It*, *Explore It*, and *Explain It* activities, and to explore Examples.
- VIDEOS** Watch clips to support 3-Act Mathematical Modeling Lessons and STEM Projects.
- PRACTICE** Practice what you've learned.

### 3-ACT MATH



#### Mixin' It Up

Drinking plenty of water each day is important. Water is necessary for everything your body does. Not drinking enough water can lead to health problems. It's even easier to drink enough water if you like the taste. There are many ways to make water more exciting. You can drink seltzer or filtered water. You can add fruit, vegetables, herbs, or flavor enhancers. You can add more or less based on what you like. Think about this during the 3-Act Mathematical Modeling lesson.



#### Additional Digital Resources

- TUTORIALS** Get help from *Virtual Nerd*, right when you need it.
- KEY CONCEPT** Review important lesson content.
- GLOSSARY** Read and listen to English/Spanish definitions.
- ASSESSMENT** Show what you've learned.

- MATH TOOLS** Explore math with digital tools.
- GAMES** Play Math Games to help you learn.
- ETEXT** Interact with your Student's Edition online.

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## Topic Essential Question

How can you recognize and represent proportional relationships and use them to solve problems?

Revisit the Topic Essential Question throughout the topic. See the Teacher's Edition for the Topic Review for notes about answering the question.

## 3-Act Mathematical Modeling

Have students read about the Mathematical Modeling lesson for this topic. You can use the preview for this lesson to get students interested in learning the content of this topic.

The Mathematical Modeling in 3 Acts lesson appears after Lesson 2-4.



Video

Activity

# 3-ACT MATH

## 3-Act Mathematical Modeling: Mixin' It Up

### Lesson Overview

#### Objective

Students will be able to:

- ✓ use mathematical modeling to represent a problem situation and to propose a solution.
- ✓ test and verify the appropriateness of math models.
- ✓ explain why the results from mathematical models may not align exactly to the problem situation.

#### Essential Understanding

Many real-world problem situations can be represented with a mathematical model, but that model may not represent a real-world situation exactly.

Earlier in this topic, students:

- used equivalent ratios to understand and describe proportional relationships.

In this lesson, students:

- develop a mathematical model to represent and propose a solution to a problem situation involving proportions.

Later in this course, students will:

- refine their mathematical modeling skills.

This mathematical modeling lesson focuses on **application** of both **math content** and **math practices and processes**.

- Students draw on their understanding of ratio and proportionality concepts to develop a representative model.
- Students apply their mathematical model to test and validate its applicability to similar problem situations.

### Math Anytime



#### Today's Challenge

Use the Topic 2 problems any time during this topic.

Go back

Today's Challenge

Day 1 2 3 4 5 DIY

**Model**  
Choose one of the elevators listed in the table. Write the speed this elevator travels in 1 second as a rate.

Building	Approximate Distance (feet)	Time (seconds)
Empire State Building	233	10
John Hancock Center	150	5
One World Trade Center	666	20
Space Needle	368	25
Willis Tower	401	15

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### Mathematics Overview

In this lesson, students will develop and use a mathematical model to represent and propose a solution to a real-world problem involving proportions. Students will reinforce their procedural skills as well as their understanding of the limitations of some mathematical models for real-world situations.

#### Applying Math Practices

##### Model with Math

The focus of this lesson is on mathematical modeling. Students identify the relationship among variables, develop a model that represents the situation, and use the model to propose a solution.

Students interpret their solutions and propose explanations for why their answers may not match the real-world answer.

As students carry out mathematical modeling, they will also engage in sense-making, abstract and quantitative reasoning, and mathematical communication and argumentation. In testing and validating their models, students look for patterns and structure.



# 3-Act Mathematical Modeling

## ACT 1 The Hook



Students will be tasked with making the liquid in a water glass and a large water cooler have the same flavor.

### Play the Video and Brainstorm Questions

Have students complete **Question 1**. Encourage them to consider the situation and ask any questions that arise. Listen for interesting mathematical and non-mathematical questions. Ask students what makes each question interesting.

**Q: What questions do you have?** [Sample questions: Who is going to drink all of that water? How much water is in the cooler? What is being added to the water? Why is it being added?]

### Pose the Main Question

After the question brainstorming, pose the Main Question students will be tasked with answering. Have students complete **Question 2**.

#### Main Question

**Q: How many drops are needed for the water in the cooler to have the same flavor as the water in the glass?**

### Ask about Conjectures

Have students complete **Questions 3–5**. You can survey the class for the range of predictions.

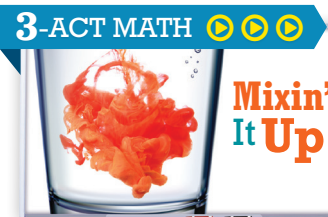
**Q: Why do you think your conjecture is the solution to the Main Question?**

**Q: Who had a similar conjecture?**

**Q: How many of you agree with that conjecture?**

**Q: Who has a different conjecture?**

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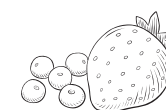
**3-Act Mathematical Modeling:**  
**Mixin' It Up**  
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#### ACT 1

1. After watching the video, what is the first question that comes to mind?

2. Write the Main Question you will answer.

3. **Construct Arguments** Predict an answer to this Main Question. Explain your prediction.



4. On the number line below, write a number that is too small to be the answer. Write a number that is too large.



5. Plot your prediction on the same number line.

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Activity

# 3-Act Mathematical Modeling *continued*

## ACT 2 The Model



### Identify Variables

Have students complete **Questions 6 and 7**.

**Q:** What information do you need to know to solve the problem? [Sample answers: How much water is in the glass; how many drops are in the glass; how much water is in the water cooler.]

**Q:** How could you get that information?

**Q:** Why do you need that information?

### Reveal the Information

As students identify needed information, you can use the online interactivity to estimate and then reveal information. Alternatively, you can share the information provided below.

Amount of water in the glass: 12 ounces

Number of drops added to glass: 5

Size of water cooler: 10 gallons

1 gallon = 128 ounces

### Develop a Model

As students answer **Questions 8 and 9**, look for inefficient methods that they are using and prompt them to think about more efficient solutions.

For example, a student may create the following table:

Amount of Water	Number of Drops
12 oz	5
24 oz	10
36 oz	15
48 oz	20

**Q:** How can you show the relationship between the amount of water and the number of drops more efficiently? [Sample answer: Use a ratio.]

### Use the Model to Propose a Solution

After students answer **Questions 8 and 9**, use the Possible Student Solutions below as you facilitate a discussion about solution methods.

#### Possible Student Solutions

##### Marco's Work

1 gallon = 128 oz, so 10 gallons = 1,280 gallons

$$\frac{5 \text{ drops}}{12 \text{ oz}} = \frac{d \text{ drops}}{1,280 \text{ oz}}$$

$$\frac{5}{12} \times 1,280 = \frac{d}{1,280} \times 1,280$$

$$533 \approx d$$

The water cooler needs 533 drops.

**Marco** uses equivalent ratios to write and solve a proportion.

##### Vicki's Work

$$\text{unit rate: } \frac{12 \text{ oz}}{5 \text{ drops}} = 2.4 \text{ ounces per drop}$$

$$w = 2.4d$$

w is water in ounces

d is number of drops

10 gallons is  $10 \times 128$  ounces

$$w = 2.4d$$

$$w = 2.4(125 \times 10)$$

$$w \approx 533$$

**Vicki** uses the constant of proportionality to write and solve a different form of the same proportion.

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#### ACT 2

**6.** What information in this situation would be helpful to know? How would you use that information?



**7. Use Appropriate Tools** What tools can you use to get the information you need? Record the information as you find it.

**8. Model with Math** Represent the situation using the mathematical content, concepts, and skills from this topic. Use your representation to answer the Main Question.

**9.** What is your answer to the Main Question? Is it higher or lower than your prediction? Explain why.



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Video

## ACT 3 The Solution and Sequel



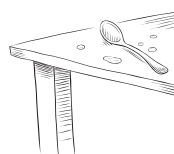
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### ACT 3

10. Write the answer you saw in the video.

11. **Reasoning** Does your answer match the answer in the video? If not, what are some reasons that would explain the difference?

12. **Make Sense and Persevere** Would you change your model now that you know the answer? Explain.



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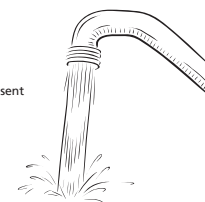
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### ACT 3 Extension

#### Reflect

13. **Model with Math** Explain how you used a mathematical model to represent the situation. How did the model help you answer the Main Question?

14. **Critique Reasoning** Choose a classmate's model. How would you adjust that model?



### SEQUEL

15. **Use Structure** A classmate usually adds 6 drops to 16 ounces of water. Use your updated model to predict the number of drops she would use for the large container.



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### Use the Video to Reveal the Answer

The final part of the video shows the entire process of adding flavoring to the water cooler. Have students complete **Question 10**. Congratulate the students who were closest to the actual answer.

#### Main Question Answer

518 drops

### Validate Conclusions

After students complete **Questions 11 and 12**, encourage them to discuss possible sources of error inherent in using math to model real-world situations. Look for students to point out that their models are still useful even though they are not perfect.

**Q:** Why does your answer not match the answer in the video?  
[Sample answer: The video shows that the water cooler is not completely full.]

**Q:** How useful was your model at predicting the answer?

**Q:** How could your model better represent the situation?

### Reflect on Thinking

**Critique Reasoning** If time allows, have students complete **Questions 13 and 14** as an extension. Use this opportunity to discuss how students incorporate mathematical processes during the task.

### Pose the Sequel

**Use Structure** Use **Question 15** to present a similar problem situation involving proportions so that they can test the usefulness of their models.

**Q:** A classmate usually adds 6 drops to 16 ounces of water. Use your updated model to predict the number of drops she would use for the water cooler.

Using their models and the answer in the video, look for student solutions between 450 and 480 drops.

**Q:** How does the flavor of this water compare to the flavor of the water in the video? [This water has a weaker flavor.]